# Throughput Optimization in OFDMA Cognitive Radio Networks Based on Node Selection and Power Allocation

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**Abstract.** Node selection and resource management are two key issues in cognitive radio networks. Due to channel impairments in wireless networks, cooperative spectrum sensing is used in orthogonal frequency division multiple access (OFDMA) cognitive radio networks. The problem is designed to maximize the system throughput under the constraints of power budget and detection performance. This problem is solved using convex optimization methods and the priority of nodes for participating in sensing and transmission processes and also the optimal power of transmission nodes are obtained. For implementing this solution, three algorithms are proposed. The results reveal the benefits of the proposed algorithms in terms of throughput and sensing performance.

**Keywords:** Cognitive radio; cooperative spectrum sensing; OFDMA; throughput; power allocation.

## 1. Introduction

Due to inefficient use of the spectrum in static spectrum access which allocates the spectrum to licensed users with sparsely utilization in temporal and geographical dimensions, dynamic spectrum access (DSA) has been introduced as the promising solution for efficient utilization of the scarce radio resources.

Cognitive radio (CR) based on DSA has emerged as an intelligent wireless technology to expand the spectrum utilization by allowing the unlicensed or secondary users (SUs) to access the spectrum opportunities without affecting the transmissions of licensed or primary users (PUs) [1], [2].

CR relies on spectrum sensing to search for unused spectrum which is known as the spectrum hole. In spectrum sensing, probability of detection and probability of false alarm are two main metrics which are used for evaluation the detection performance. The probability of detection means the probability of correct detection the PU's presence when it is active actually. The probability of false alarm denotes the probability of detection the PU's presence when the band is free actually.

Manuscript received February 2, 2014; revised July 22, 2014; accepted August 15, 2014.
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Due to user uncertainty and channel impairments such as multipath fading and shadowing, cooperation with other users is employed for increasing the accuracy of sensing process and improving the detection performance. Cooperative spectrum sensing is done in two phase. In the first one, several secondary users sense the spectrum and check the presence of the primary user individually. In the second phase, all of the cooperative users send their results to fusion center (FC) for making the final decision about the presence of primary users [3], [4]. One of the most important factors affecting the performance of cooperative sensing, is the proper choice of users participating in the sensing of specified band [5].

In [6], it has been shown that employing a certain number of users with higher signal to noise ratio (SNR) rather than using all of the users for cooperation, achieves the optimum values of probability of detection and probability of false alarm.

The authors in [7] choose the sensing users based on detection performance. For this purpose, three methods Simple Counting (SC), Partial-Agreement Counting (PAC), and Collision Detection (CD) are proposed. In SC, the users with the higher number of PU's presence decision, are selected for cooperation. In PAC, the users with higher agreement with FC are selected and finally in CD, the users with higher number of correct PU's detection when FC decides wrongly, are selected for cooperation. In [6] the authors propose practical sensing node selection for cooperative spectrum sensing in the case that malicious users are existed in the network. To achieve this goal, they define the consistency metric to determine the consistencies between the sensing results of unknown SUs and trusted SUs. The SUs with the highest value of this metric are used for cooperation.

After determining the unused frequency bands in sensing process, the efficient transmission technique is required to fill these free bands. To achieving this goal, OFDM technique is an appropriate candidate for cognitive radio networks. In OFDM, the band is divided into several narrower bands which called subcarriers. All of the subcarriers in OFDM are employed for transmitting the symbols of a single user. Thus, OFDM cannot be used for resource sharing among multi users. So, it should be combined with multiple access techniques such as TDMA (OFDM-TDMA), FDMA (OFDMA) and CDMA (OFDMA-CDMA) to combat this issue [8]. Among these techniques, OFDMA is most common. In OFDMA, a subcarrier (or a group of subcarriers) can be assigned to each one of the users based on their priority. By using OFDMA in cognitive radio networks, the occupied subcarriers can be modulated with zero and the appropriate signal can be adaptively shaped for avoiding the interference to PUs. One of the other advantages of OFDMA in CR networks is its flexibility in resource allocation. Many works address the problem of power and subcarrier allocation in OFDMA CR networks to achieve system improvement in terms of throughput, battery life, interference, and so on. In subcarrier allocation, by benefiting from independent channel condition for multi users (*i.e.* multiuser diversity), each subcarrier or sub channel (a group of subcarriers) can be assigned to the user with the highest priority. In power allocation, the power level for each subcarrier is obtained to optimize the specified objective function [9].

In [10], the power level is proportional to the SNR of subcarriers. This method is called water filling that allocates more power to the subcarriers with higher SNR to maximize the system throughput subject to the constraint of total transmit power.

According to [11], water filling method is not appropriate for power allocation in OFDM system because the protection of PUs is not considered. Thus, the optimization problem is designed that keeps the sum of the interference power to PUs, below the certain constant threshold. Then three algorithms, Max-Rate subcarrier allocation algorithm (MaxR-SAA), Min-Interference subcarrier allocation algorithm (MinI-SAA) and Fair-Rate subcarrier allocation algorithm (FairR-SAA), are proposed. After that, optimal power of each subcarrier is calculated and compared with three suboptimal algorithms named distance based step power allocation, interference based step power allocation, and modified equal power allocation. But this paper separates the subcarrier and power allocation and do them in two stages.

In [12], the interference threshold is considered variable and is obtained in terms of primary links SNR in two cases perfect channel state information (CSI) and imperfect CSI. Then the authors present an optimal algorithm for jointly subcarrier and power assignment in which the priority of users for subcarrier allocation is determined based on the compromise between capacity and fairness and the power of users in each subcarrier is obtained via solving the problem with Lagrange method.

In aforementioned papers, the authors do not consider spectrum sensing and only the power and subcarrier allocation have been discussed for given available frequency bands for secondary users.

In [13], the authors aim at finding the optimal power and spectrum sensing time for spectrum efficiency maximization under the constraints of average BER threshold, total transmit power budget, probability of detection and probability of false alarm in the ultra-wideband cognitive radio networks. For solving this problem, the group power allocation algorithm is used in which the subcarriers are classified into groups and the power is loaded on these groups by greedy algorithm at first. Then the bits are assigned on the subcarriers in each subcarrier group by equal power allocation. More, the authors prove that finding the optimal spectrum sensing time is very complex, hence the numerical method is employed to find this optimum value.

In [14], first the optimum number of SUs for cooperation is obtained to minimize the error probability. Then it is shown that for specified sensing time, the throughput has been maximized and also, the maximum value of throughput is decreasingly function in terms of the optimum number of cooperative SUs. But in this paper, single channel setting is considered and also it is not determined which SUs participate in sensing the channel.

The purpose of this paper is to select the appropriate nodes for sensing and transmission operations and also assign the optimal power level to the transmission nodes, in order to maximize the system throughput with consideration of detection performance and total power budget constraints. In this work, energy detection scheme is applied for identifying the existence of primary users in licensed bands because it does not need to any prior knowledge and has low complexity in computation and implementation. Also, OR fusion rule is employed for combining the decisions of cooperating SUs in FC.

The rest of the paper is organized as follows. In Section 2, system model is expressed. Section 3 brings up the problem formulation and also the mathematical solution of this problem is discussed in this section. Section 4 presents our proposed algorithm for solving the problem. The performance of our algorithm will be shown in simulation results in Section 5. Finally, the conclusion is presented in Section 6.

## 2. System Model

We consider a cognitive radio network with $N$ secondary users, a FC, a primary base station(PU-BS),a secondary base station(SU-BS) and $K$ orthogonal frequency bands (OFDM subcarriers). Each subcarrier is licensed for a PU(hence there are $K$ PUs).

At first, the SUs should sense the subcarriers to know which one of them is free. As mentioned before, the energy detection scheme is used for detecting the free subcarriers. For this purpose, the energy of received signal should be calculated in each one of sensing users.

If $y_{k,n}(i)$ be the discrete received signal of subcarrier $k$ that observed by node $n$, two hypotheses is defined as follows:

$$H_0 : y_{k,n}(i) = u_{k,n}(i) \quad i = 1,...,\delta f_s \tag{1}$$

This hypothesis suggests that the primary user is not present in the band and thus the received signal is equal to channel noise. In (1), $u_{k,n}$ is the independent and identically distributed (i.i.d) Gaussian noise on subcarrier $k$ which has zero mean and variance $\sigma_u^2$ . $\delta$ is the sensing time duration, $f_s$ is sampling frequency, hence $\delta f_s$ denotes the number of samples.

When the primary user is present in the band, the second hypothesis will be expressed as follows:

$$H_1 : y_{k,n}(i) = h_{k,n}s_{k,n}(i) + u_{k,n}(i) \quad i = 1,...,\delta f_s \tag{2}$$

where $s_{k,n}$ is a primary signal on the subcarrier $k$ that observed by sensing node $n$ and it is assumed to be a i.i.d random process with zero mean and variance $\sigma^2_{s_{k,n}}$ . Also, $s_{k,n}$ and $u_{k,n}$ are assumed independent from each other. $h_{k,n}$ is the channel gain between primary user $k$ and node $n$ which is modeled due to path loss and shadowing effects in this work.

$$h_{k,n} = 10^{\frac{L_{k,n}}{20}} \tilde{h}_{k,n} \tag{3}$$

where $\tilde{h}_{k,n}$ is complex Gaussian random process (Rayleigh fading) with zero mean and variance 1. $L_{k,n}$ includes the path loss and shadowing components. The free space path loss component of $L_{k,n}$ for a line of sight (LOS) channel is given by:

$$l^{PL}_{k,n} = (\frac{\sqrt{G}\lambda}{4\pi d_{k,n}})^2 \tag{4}$$

Here, $\lambda$ is the wavelength and equal to $\frac{c}{f_c}$ that $c$ is speed of light and $f_c$ is carrier frequency, $G$ is the product of the transmit and receive antenna gain. In our work we assume $G = 1$ , $d_{k,n}$ is the distance between primary user $k$ and node $n$. According to (4), $L_{k,n}$ is obtained as follows:

$$L_{k,n} = 10\log(l^{PL}_{k,n}) + X \tag{5}$$

$X$ is a Gaussian random variable (in dB) with zero mean and variance $\sigma^2_X$ that models the shadowing effect.

Under the hypothesis $H_1$, the signal to noise ratio (SNR) of primary user $k$ that measured by node $n$ is defined as follows:

$$\gamma_{k,n} = \frac{|h_{k,n}|^2 \sigma^2_{s_{k,n}}}{\sigma^2_u} \tag{6}$$

Similarly the SU uplink channel is modeled as follows:

$$h^{UP}_{k,n} = 10^{\frac{L^{UP}_n}{20}} \tilde{h}^{UP}_{k,n} \tag{7}$$

Similarly $\tilde{h}^{UP}_{k,n}$ is a Rayleigh fading part with zero mean and variance 1.The shadowing and path loss components are modeled in $L^{UP}_n$ which is expressed as follows.

$$L^{UP}_n = 20\log(\frac{\lambda}{4\pi d_n}) + X^{UP} \tag{8}$$

where $d_n$ is denotes the distance between node $n$ and SU-BS. $X^{UP}$ is a Gaussian random variable with zero mean and variance $\sigma^2_{X^{UP}}$ and denotes the shadowing effect [15], [16].

The energy of received signal in node $n$ is given by:

$$E_{y_{k,n}} = \frac{1}{\delta f_s} \sum_{i=1}^{\delta f_s} |y_{k,n}(i)|^2 \tag{9}$$

This energy is compared with the specified threshold ε and based on this, one bit hard decision $D_{k,n}$ is sent to FC for making decision on PU's presence. $D_{k,n} = 1$ represents the PU's presence in subcarrier $k$ and $D_{k,n} = 0$ indicates that PU is absent in subcarrier $k$ .

$$\begin{cases} H_1 : E_{y_{k,n}} > \varepsilon \rightarrow D_{k,n} = 1 & (10) \\ H_0 : E_{y_{k,n}} < \varepsilon \rightarrow D_{k,n} = 0 & (11) \end{cases}$$

For a large number of samples, $E_{y_{k,n}}$ is estimated with Gaussian distribution. The local probability of detection and local probability of false alarm are obtained as follows [17]:

$$P_{f_{k,n}} = P(E_{y_{k,n}} > \varepsilon \mid H_0) = Q((\frac{\varepsilon}{\sigma^2_u} - 1)\sqrt{\delta f_s}) \tag{12}$$

$$P_{d_{k,n}} = P(E_{y_{k,n}} > \varepsilon \mid H_1) = Q((\frac{\varepsilon}{\sigma^2_u} - \gamma_{k,n} - 1)\sqrt{\frac{\delta f_s}{2\gamma_{k,n}+1}}) \tag{13}$$

The decisions of SUs are combined at FC using OR decision rule. Due to this rule, if at least one out of N secondary users report the PU's presence, the final decision will be based on spectrum occupancy. Otherwise, the specified band is assumed free. Since all SUs don't participate in sensing process, we consider an index to determine the priority of SUs for sensing each one of subcarriers. We use $\rho_{k,n}$ as assignment index of sensing. if $\rho_{k,n} = 1$, node $n$ is participated in sensing the subcarrier $k$ and $\rho_{k,n} = 0$ denotes that node $n$ does not sense the subcarrier $k$ . Thus, $P_{d_k}$ and $P_{f_k}$ of global decision for subcarrier $k$ are given by:

$$P_{d_k} = 1 - \prod_{n=1}^{N} (1 - \rho_{k,n} P_{d_{k,n}}) \quad k = 1,...,K \tag{14}$$

$$P_{f_k} = 1 - \prod_{n=1}^{N} (1 - \rho_{k,n} P_{f_{k,n}}) \quad k = 1,...,K \tag{15}$$

## 3. Problem Formulation

One of the performance metrics of cognitive radio networks is the throughput of secondary users which is defined as the number of bits per second of SUs that are physically delivered successfully. the transmission rate of node $n$ on subcarrier $k$ is obtained via Shannon capacity formula.

$$R_{k,n} = \log_2\left(1+\frac{\left|h_{k,n}^{UP}\right|^2 p_{k,n}}{\sigma_u^2}\right) \tag{16}$$

where $p_{k,n}$ is transmit power of node $n$ on subcarrier $k$, $\sigma_u^2$ is variance of the noise.

The system throughput is sum of the transmission rates of users on all of subcarriers and is given by:

$$R = \sum_{k=1}^{K}\sum_{n=1}^{N} \psi_{k,n} \log_2\left(1+\frac{\left|h_{k,n}^{UP}\right|^2 p_{k,n}}{\psi_{k,n}\sigma_u^2}\right) \tag{17}$$

$\psi_{k,n}$ is the assignment index for transmission and indicates which user transmits on each subcarrier. $\psi_{k,n}=1$ means that the user $n$ is selected for transmission on subcarrier $k$ and $\psi_{k,n}=0$ means that the user $n$ doesn't occupy the subcarrier $k$.

In this problem optimization, our goal is choosing the appropriate nodes for participating in cooperative sensing of each channel and also allocating subcarrier and power jointly to the transmission nodes so that the system throughput is maximized while the constraints on the global probability of detection and global probability of false alarm are satisfied and the transmission power not exceeding the available power budget. Therefore, the problem formulation can be written as follows:

$$\max_{\psi_{k,n},\rho_{k,n},p_{k,n}} R = \sum_{k=1}^{K}\sum_{n=1}^{N} \psi_{k,n} \log_2\left(1+\frac{\left|h_{k,n}^{UP}\right|^2 p_{k,n}}{\psi_{k,n}\sigma_u^2}\right)$$

$$S.t.$$

$$\sum_{k=1}^{K}\sum_{n=1}^{N} \psi_{k,n} p_{k,n} \le P_T \tag{18.1}$$

$$P_{f_k} \le \alpha \quad \forall k \in \{1,2,...,K\} \tag{18.2}$$

$$P_{d_k} \ge \beta \quad \forall k \in \{1,2,...,K\} \tag{18.3}$$

$$\sum_{k=1}^{K} \rho_{k,n} \le 1 \quad \forall n \in \{1,2,...,N\} \tag{18.4}$$

$$\sum_{k=1}^{K}\sum_{n=1}^{N} \psi_{k,n}\rho_{k,n} = 0 \tag{18.5}$$

$$\sum_{n=1}^{N} \psi_{k,n} = 1 \quad \forall k \in \{1,2,...,K\} \tag{18.6}$$

$$p_{k,n} \ge 0,\ \psi_{k,n} \in \{0,1\} \ \ \rho_{k,n} \in \{0,1\} \tag{18.7}$$

where $P_T$ is the total power budget. (18.2) and (18.3) are the detection performance constraints. Smaller value of $\alpha$ provides higher opportunities for reusing the spectrum when it is free and bigger $\beta$ provides better protection of PUs. (18.4) denotes that each user can sense one subcarrier or may not participate in the sensing of any subcarriers. (18.5) shows that each user cannot participate in sensing and transmission simultaneously. Equation (18.6) denotes that each subcarrier is allocated to only one user for transmission.

According to (12) and (15), $P_{f_k}$ is independent of $\gamma_{k,n}$. Therefore. (18.2) can be replaced with another constraint which is obtained as:

$$\ln(1-\alpha) \le \ln\left(\prod_{n=1}^{N}(1-\rho_{k,n}P_{f_{k,n}})\right) \tag{19}$$

$$\ln(1-\alpha) \le \sum_{n=1}^{N} \ln\left(1-\rho_{k,n} Q\left(\left(\frac{\varepsilon}{\sigma_u^2}-1\right)\sqrt{\delta f_s}\right)\right) \tag{20}$$

$$n_k \le \left\lfloor \frac{\ln(1-\alpha)}{\ln\left(1-Q\left(\left(\frac{\varepsilon}{\sigma_u^2}-1\right)\sqrt{\delta f_s}\right)\right)} \right\rfloor = M \tag{21}$$

where $n_k = \sum_{n=1}^{N}\rho_{k,n}$ is the number of participating nodes in sensing the subcarrier $k$. $M$ denotes the maximum number of participating nodes in sensing process of each subcarrier.

This problem is an integer programming problem because of integer nature of $\psi_{k,n}$ and $\rho_{k,n}$. Hence, this problem is a non-convex problem and finding the solution is very complex. For solving this problem, we relax the problem with assuming that $\psi_{k,n}$ and $\rho_{k,n}$ are continuous parameters in the range from 0 to 1 [18]. Also, we convert the problem into standard format:

$$\min_{\psi_{k,n},\rho_{k,n},p_{k,n}} f = -\sum_{k=1}^{K}\sum_{n=1}^{N} \psi_{k,n} \log_2\left(1+\frac{\left|h_{k,n}^{UP}\right|^2 p_{k,n}}{\psi_{k,n}\sigma_u^2}\right)$$

$$S.t.$$

$$\sum_{k=1}^{K}\sum_{n=1}^{N} \psi_{k,n} p_{k,n} - P_T \le 0 \tag{22.1}$$

$$\sum_{n=1}^{N} \rho_{k,n} - M \le 0 \quad \forall k \in \{1,2,...,K\} \tag{22.2}$$

$$\beta - \left(1-\prod_{n=1}^{N}(1-\rho_{k,n}P_{d_{k,n}})\right) \le 0 \quad \forall k \in \{1,2,...,K\} \tag{22.3}$$

$$\sum_{k=1}^{K}\rho_{k,n}-1\leq 0\ \ \forall n\in\{1,2,...,N\} \tag{22.4}$$

$$\sum_{k=1}^{K}\sum_{n=1}^{N}\psi_{k,n}\rho_{k,n}=0 \tag{22.5}$$

$$\sum_{n=1}^{N}\psi_{k,n}-1=0\ \ \forall k\in\{1,2,...,K\} \tag{22.6}$$

$$-p_{k,n}\leq 0\ \ \psi_{k,n}\in(0,1)\ \ \rho_{k,n}\in(0,1) \tag{22.7}$$

To check the convexity of problem, the Hessian of objective function ($\nabla^2 f$) is obtained as follows:

$$\nabla^2 f=\frac{p_{k,n}\left(\dfrac{|h_{k,n}|^2}{\sigma_u^2}\right)^2}{\ln 2\left(\psi_{k,n}+p_{k,n}\dfrac{|h_{k,n}|^2}{\sigma_u^2}\right)^2}\begin{bmatrix}\dfrac{p_{k,n}}{\psi_{k,n}} & 1\\ 1 & \dfrac{\psi_{k,n}}{p_{k,n}}\end{bmatrix} \tag{23}$$

The problem is convex because $\nabla^2 f$ is positive semidefinite matrix. Also the constraints are convex, hence, Slater condition is satisfied and we can use Karush-Kuhn-Tucker (KKT) conditions to solve this problem [19]. Considering the Lagrange multipliers $\lambda$, $\eta_k$, $\zeta_k$, $\vartheta_n$, $\omega$, $\mu_k$, and $\theta_{k,n}$ for constraints (22.1), (22.2), (22.3), (22.4), (22.5), (22.6) and (22.7), respectively, we form Lagrange function as follows:

$$\begin{aligned}&L(\psi_{k,n},\rho_{k,n},p_{k,n},\lambda,\eta_k,\zeta_k,\omega,\vartheta_n,\mu_k)=\\&-\sum_{k=1}^{K}\sum_{n=1}^{N}\psi_{k,n}\log_2(1+\frac{|h_{k,n}^{UP}|^2\,p_{k,n}}{\psi_{k,n}\sigma_u^2})+\lambda(\sum_{k=1}^{K}\sum_{n=1}^{N}\psi_{k,n}p_{k,n}-P_T)\\&+\sum_{k=1}^{K}\eta_k(\sum_{n=1}^{N}\rho_{k,n}-M)+\sum_{k=1}^{K}\zeta_k(\beta-(1-\prod_{n=1}^{N}(1-\rho_{k,n}P_{d_{k,n}})))\\&+\omega\sum_{k=1}^{K}\sum_{n=1}^{N}\psi_{k,n}\rho_{k,n}+\sum_{n=1}^{N}\vartheta_n(\sum_{k=1}^{K}\rho_{k,n}-1)\\&+\sum_{k=1}^{K}\mu_k(\sum_{n=1}^{N}(\psi_{k,n}-1))-\sum_{k=1}^{K}\sum_{n=1}^{N}\theta_{k,n}p_{k,n}\end{aligned} \tag{24}$$

KKT conditions reveal that the optimal values of $\rho_{k,n}^*$, $\psi_{k,n}^*$, $p_{k,n}^*$ are obtained through differentiating $L$ with respect to $\rho_{k,n}$, $\psi_{k,n}$, $p_{k,n}$, and setting the derivatives to zero, respectively.

$$\begin{aligned}\frac{\partial L}{\partial\rho_{k,n}}&=\eta_k-\zeta_k P_{d_{k,n}}\prod_{i\neq n}(1-\rho_{k,i}P_{d_{k,i}})+\omega\psi_{k,n}\\&+\sum_{n=1}^{N}\vartheta_n=0\end{aligned} \tag{25}$$

Solving this equation in terms of $\rho_{k,n}$ is very difficult. It is proved in [20] that obtaining the ratio $\rho_{k,n}/\rho_{k,j}$ through mathematical operations, can be used to define a criterion to determine the user's priority for sensing.

$$\frac{\rho_{k,n}}{\rho_{k,j}}=\frac{\eta_k+\omega\psi_{k,n}+\sum_j\vartheta_j-\zeta_k P_{d_{k,j}}\prod_{i\neq n,j}(1-\rho_{k,i}P_{d_{k,i}})}{\eta_k+\omega\psi_{k,n}+\sum_n\vartheta_n-\zeta_k P_{d_{k,n}}\prod_{i\neq n,j}(1-\rho_{k,i}P_{d_{k,i}})} \tag{26}$$

$$\text{cost}(n)=\eta_k-\zeta_k P_{d_{k,n}}\prod_{i\neq n,j}(1-\rho_{k,i}P_{d_{k,i}})+\omega\psi_{k,n}+\sum_{n=1}^{N}\vartheta_n \tag{27}$$

Since the sensing nodes cannot participate in transmission process, $\psi_{k,n}=0$. Also, the ratio of $\rho_{k,n}/\rho_{k,j}$ is inversely related to the ratio of the equivalent expression of each one. Therefore, the user with lower cost($n$) in (27), has higher priority for sensing. $\sum_{n=1}^{N}\vartheta_n$ is constant for all of $n$ values and can be removed from (27).

For obtaining the priority of nodes for transmission, we have:

$$\begin{aligned}\frac{\partial L}{\partial\psi_{k,n}}&=-\frac{\ln\left(1+\dfrac{|h_{k,n}^{UP}|^2\,p_{k,n}}{\psi_{k,n}\sigma_u^2}\right)}{\ln(2)}+\frac{\left(\dfrac{|h_{k,n}^{UP}|^2\,p_{k,n}}{\psi_{k,n}\sigma_u^2}\right)}{\ln(2)\left(1+\dfrac{|h_{k,n}^{UP}|^2\,p_{k,n}}{\psi_{k,n}\sigma_u^2}\right)}\\&+\lambda\,p_{k,n}\,\omega\rho_{k,n}+\sum_k\mu_k=0\end{aligned} \tag{28}$$

Considering $x=1+\dfrac{p_{k,n}|h_{k,n}|^2}{\psi_{k,n}\sigma_u^2}$ and $c=(\ln 2)(\lambda p_{k,n}+\omega\rho_{k,n}+\sum_k\mu_k)$, we have [21]

$$-x^{-1}e^{-x^{-1}}=-e^{-(1+c)} \tag{29}$$

The solution of this equation is given by

$$-x^{-1}=W(-e^{-(1+c)}) \tag{30}$$

where $W(.)$ is Lambert function

$$\frac{|h_{k,n}^{UP}|^2\,p_{k,n}}{\psi_{k,n}\sigma_u^2}=e^{(1+W(-\exp(-(1+c)))+c)} \tag{31}$$

As seen in (31), $\frac{\left|h_{k,n}^{UP}\right|^2 p_{k,n}}{\psi_{k,n}\sigma_u^2}$ is constant. Hence, for maximizing $\psi_{k,n}$, $\frac{\left|h_{k,n}^{UP}\right|^2 p_{k,n}}{\psi_{k,n}\sigma_u^2}$ should be maximized. It means that each subcarrier is allocated to the user which has maximum value of $\frac{\left|h_{k,n}^{UP}\right|^2 p_{k,n}}{\psi_{k,n}\sigma_u^2}$ in that subcarrier.

$$n = \arg\max_n \left(\frac{\left|h_{k,n}^{UP}\right|^2 p_{k,n}}{\sigma_u^2}\right) \tag{32}$$

the optimum value of power in each subcarrier, is obtained as follows:

$$\frac{\partial L}{\partial p_{k,n}} = \frac{\left|h_{k,n}^{UP}\right|^2 / \sigma_u^2}{\ln(2)\left(1+\frac{\left|h_{k,n}^{UP}\right|^2 p_{k,n}}{\psi_{k,n}\sigma_u^2}\right)} + \lambda\psi_{k,n} - \theta_{k,n} = 0 \tag{33}$$

By solving this equation in terms of $p_{k,n}$, we have

$$p_{k,n} = \frac{\psi_{k,n}}{\ln(2)(\lambda\psi_{k,n})} - \frac{\psi_{k,n}\sigma_u^2}{\left|h_{k,n}^{UP}\right|^2} \tag{34}$$

Power is calculated only for users which are used for transmission, so $\psi_{k,n} = 1$ in (34).

To obtain the optimum values of Lagrange multipliers in (27), (32) and (34), complimentary slackness conditions should be analyzed as follows:

$$\eta_k^*\left(\sum_{n=1}^{N}\rho_{k,n}^* - M\right) = 0 \quad \begin{cases} \eta_k^* > 0 \quad \sum_{n=1}^{N}\rho_{k,n}^* = M & (35.1) \\ \eta_k^* = 0 \quad \sum_{n=1}^{N}\rho_{k,n}^* < M & (35.2) \end{cases}$$

$$\zeta_k^*(\beta - P_{d_k}) = 0 \quad \begin{cases} \zeta_k^* > 0 \quad P_{d_k} = \beta & (35.3) \\ \zeta_k^* = 0 \quad P_{d_k} > \beta & (35.4) \end{cases}$$

$$\lambda^*\left(\sum_{k=1}^{K}\sum_{n=1}^{N}\psi_{k,n}^* p_{k,n}^* - P_T\right) = 0$$

$$\begin{cases} \lambda^* > 0 \quad \sum_{k=1}^{K}\sum_{n=1}^{N}\psi_{k,n}^* p_{k,n}^* = P_T & (35.5) \\ \lambda^* = 0 \quad \sum_{k=1}^{K}\sum_{n=1}^{N}\psi_{k,n}^* p_{k,n}^* < P_T & (35.6) \end{cases}$$

$$\theta_{k,n}^* p_{k,n}^* = 0 \quad \begin{cases} \theta_{k,n}^* > 0 \quad p_{k,n}^* = 0 & (35.7) \\ \theta_{k,n}^* = 0 \quad p_{k,n}^* > 0 & (35.8) \end{cases}$$

For the first condition ($\eta_k^*(\sum_{n=1}^{N}\rho_{k,n}^* - M) = 0$), each one of the two modes (35.1) or (35.2) can be occurred. Also, this is true for second condition ($\zeta_k^*(\beta - P_{d_k}) = 0$).Because $\eta_k$ and $\zeta_k$ in (27) are independent of $n$ and are identical for all of the users on the specified channel. Hence, these parameters have no effect on determining the priority of users for sensing the particular channel and cannot be considered. In the condition $\lambda^*(\sum_{k=1}^{K}\sum_{n=1}^{N}\psi_{k,n}^* p_{k,n}^* - P_T) = 0$, if $\lambda^* = 0$ is satisfied, $p_{k,n}^*$ becomes indefinite according to (34) and it is not acceptable. so, $\lambda^* > 0$ is acceptable and $\lambda^*$ should be calculated via bisection algorithm in the iterative manner for obtaining $p_{k,n}^*$ until $\sum_{k=1}^{K}\sum_{n=1}^{N}\psi_{k,n}^* p_{k,n}^* = P_T$ is satisfied.

(35.7) and (35.8) state that the power of user *n* on subcarrier *k*, cannot be lower than zero. thus, (34) can be rewritten as follows:

$$p_{k,n}^* = \left(\frac{1}{(\ln 2)\lambda^*} - \frac{\sigma_u^2}{\left|h_{k,n}^{UP}\right|^2}\right)^+ \tag{36}$$

where $(.)^+$ is equal to $\max(0,.)$. Notice that $\psi_{k,n}^* = 1$ for transmission nodes. Thus, it is removed from (34).

## 4. Proposed Algorithm

This paper presents three algorithms SSF (Sensing node selection first), TSF (transmission node selection first) and PSR (PUs selected randomly) to implement the mentioned problem. It should be noted that in the proposed algorithms, the nodes are cooperated to each other. Also, the sensing nodes and transmission nodes cannot be selected simultaneously. In SSF and PSR, the sensing nodes are chosen at first but in TSF, transmission nodes are selected at first.

In SSF algorithm we start by cooperative spectrum sensing using energy detection and continue by allocating the proper subcarrier and power for transmission. Fig1 shows the steps of this algorithm in detail. We should select the sensing nodes for cooperation in sensing process of each subcarrier at first. In this method, the PUs are arranged according to their distance from center of the region to determine their priority for assigning the sensing nodes. This is done because there are lower SUs around the PUs with greater distances from center of the region. Thus, if the sensing nodes are chosen for the closer PUs at first, the farther PUs are forced to select their sensing nodes among the SUs with great distances from themselves. This leads to reduction in $P_{d_k}$ for farther PUs. As a result, the PU with more distance has higher priority for assigning the sensing nodes. In order to selecting the sensing nodes for specified subcarrier, cost is calculated for all of the users individually and sorted in ascending order. Then, as long as the number of participating nodes in sensing of that subcarrier is not more than M, we can add the next node with the lowest cost (highest priority) to the set of sensing nodes of that subcarrier, to meet the constraint $P_{d_k} \geq \beta$ (step 2 in Fig. 1). Then, the bisection algorithm is required to obtain the optimal value of power. For this purpose, the power of all users are calculated on each subcarrier according to (34),

| SSF Algorithm |
|---|
| *Step1.* $Counter(k) = 0$ % number of assigned sensing nodes to the *k*'th subcarrier. <br> Sensing_set($k$)= $\phi$ % assigned sensing nodes to subcarrier$k$. <br> SU nodes = {*1, ..., N*} <br> *Step2.*While ( $Counter(k) < M \quad \forall k \in \{1,...,K\}$ ) <br> if $P_{d_k} \geq \beta \quad \forall k \in \{1,...,K\}$ , break, end <br> for *k=1:K* <br> Compute Distance ($k$) % distance of k'th PU. <br> Find $k_{\max}$ % the subcarrier that is licensed for the PU with highest distance from center of the region. <br> if $P_{d_{k_{\max}}} < \beta$ <br> Compute cost($n$) for set SU nodes on $k_{\max}$ <br> Determine $n_{\text{sensing}}$ %the user with the minimum value of cost($n$) . <br> *Counter(* $k_{\max}$ *)* = *Counter(* $k_{\max}$ ) +1 , Distance( $k_{\max}$ ) = 0; <br> SU nodes = SU nodes\ $n_{\text{sensing}}$ ; <br> Sensing _ set( $k_{\max}$ ) = Sensing_ set( $k_{\max}$ ) $\cup n_{\text{sensing}}$ End <br> End <br> End <br> *Step3*. $\lambda_{\min} = 0$ , $\lambda_{\max} = \varsigma$ (a large enough number) <br> While $\lvert \lambda_{\min} - \lambda_{\max} \rvert > \kappa$ (small number) <br> $\lambda = (\lambda_{\min} + \lambda_{\max})/2$ <br> for *k=1:K* <br> Compute $n_{trans}(k)$ according to (21) <br> End <br> If $\sum_{k=1}^{K} \sum_{n \in n_{trans}} p_{k,n} > P_T$ <br> $\lambda_{\min} = \lambda$ <br> else if $\sum_{k=1}^{K} \sum_{n \in n_{trans}} p_{k,n} < P_T$ <br> $\lambda_{\max} = \lambda$ <br> End <br> End |

Fig. 1. Pseudo code of SSF algorithm.

then the user with highest priority due to (32) is selected for each subcarrier and it's power will be the optimal power on that subcarrier (step 3 in Fig1).

In the second algorithm (TSF algorithm), transmission nodes and their optimal power are determined at first, then the selected nodes are removed from the set of nodes and sensing nodes are chosen among the remaining nodes. This algorithm also uses the distance based priority of PUs for assigning the sensing nodes to them. Sensing node selection in this method is the same as sensing node selection in SSF algorithm.

PSR algorithm is similar to SSF except that the PSR chooses PUs randomly for assigning the sensing nodes to them. It is clear that PSR has lower complexity compared with two other algorithms.

## 5. Simulation Results

We assume SUs and PUs are distributed uniformly in a square region with length L, FC and base stations are in the center of this region. In this work, we use 16 OFDM system, so there are 16 subcarriers which each one is licensed for a PU ($k$ = 16). The power of primary users on each subcarrier assigned is $20 \times 10^{-3}$ W. In addition, the total power budget has been assumed 20 W. the values of $\delta$ , $f_c$ , $\alpha$ , $\beta$ and $L$ are chosen as 25.5 µsec, 1.9 GHZ, 0.9, 0.1, 3000 m, respectively. Shadowing standard deviation of SU-PU link and SU uplink are assumed 3 db. Also, noise variance on each subcarrier is $10^{-11}$ W. In the simulation results, we compare simulation results in different algorithms:

1) SSF algorithm
2) TSF algorithm
3) PSR algorithm

Also, the results are obtained from averaging the results of 15000 times running the program.

Fig. 2 shows the average throughput for different number of SU nodes. It is clear that the average throughput is an increasingly function in terms of number of nodes because we have more choices for selecting the appropriate transmission nodes. In SSF algorithm, the suitable nodes for transmission may be used in sensing process. Thus, it is possible that the inappropriate nodes remain for transmission and performance of transmission is degraded. It is shown that TSF algorithm provides the highest average throughput and in PSR algorithm, the average throughput has the lowest value.

In the case that the maximum number of sensing nodes *M* is employed for a specified subcarrier and the constraint $P_{d_k} \geq \beta$ is not satisfied for that subcarrier, the problem has no answer. In Fig. 3, the success ratio is shown versus the total number of SU nodes. The success ratio is defined as the ratio of the number of cases that the problem has answer to the total number of program runs.

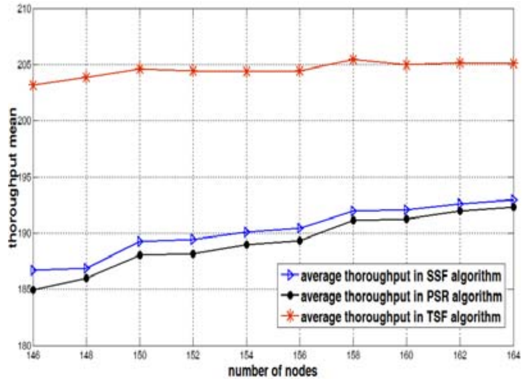


Fig. 2. Throughput mean vs the number of nodes.

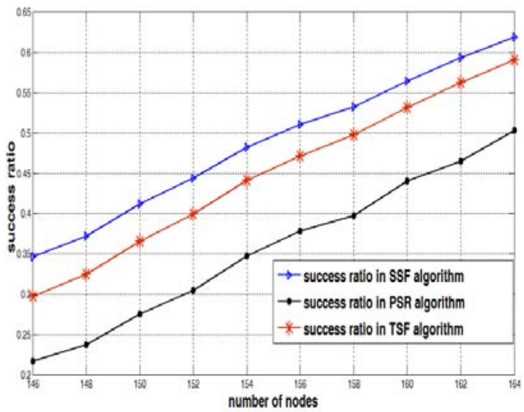


Fig. 3. Success ratio vs the number of nodes.

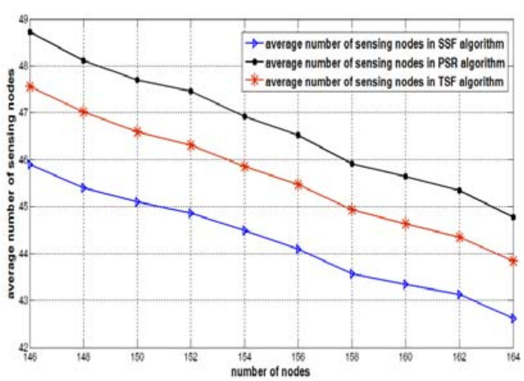


Fig. 4. Average number of employed sensing nodes vs total number of nodes.

In fact, this metric shows the ability of the algorithms for finding the feasible solutions. Increasing the number of nodes, reduces the distances between SUs and PUs and increases the values of detection probability. Therefore, the number of states that the problem has feasible solution, is increased. As a result, the success ratio is an increasing function in terms of the number of nodes. It is clear that in TSF algorithm, the success ratio is lower than SSF algorithm. Because, the sensing and transmission nodes aren't selected simultaneously. So, when we select the transmission nodes at first in TSF, the good nodes for sensing may be used for transmission process. thus, the sensing performance in TSF algorithm is lower than sensing performance in SSF.

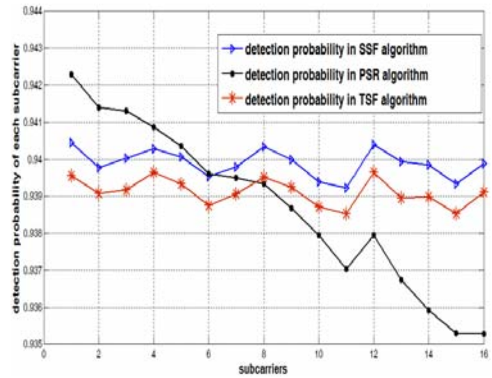


Fig. 5. Detection probability in different subcarriers for L=3000 and N=164.

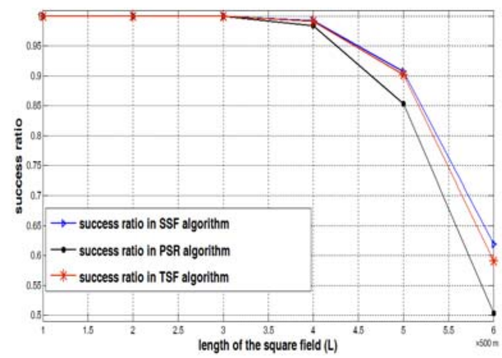


Fig. 6. Success ratio for different size of square field (N=164)

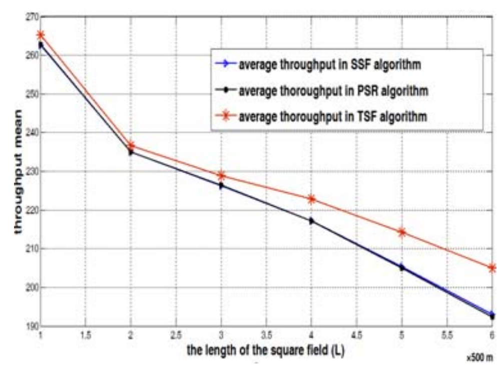


Fig. 7. Average throughput of SUs in different size of square field (N=164).

Fig. 4 presents the average number of employed sensing nodes for three cases. As mentioned above, increasing the number of nodes in a fixed size environment increases the values of detection probability. Hence, the constraint of $P_{d_k} \geq \beta$ is satisfied by employing lower number of sensing nodes. Also, we can observe that in SSF algorithm, lower number of sensing nodes are used and in PSR algorithm we need more sensing nodes than other algorithms to satisfy the constraint of detection probability $P_{d_k} \geq \beta$ .

In Fig. 5, it is shown that constraint $\mathrm{P}_{\mathrm{d}_\mathrm{k}} \geq \beta$ is satisfied for all of the subcarriers in three algorithms for N=164 and L=3000. As seen in this figure, the probability of detection in SSF algorithm is higher, compared to TSF algorithm.

Figure 6 illustrates the success ratio when the number of SUs are constant and equal to 164 and the length of the square field varies from 500 m to 3000m. Success ratio is decreased with increase of the size of the region. Because with increasing the length of the square field, the distance between the SU and PU nodes becomes higher and detection probability decreases. Also, it is clear that in TSF algorithm, the success ratio should be lower than the success ratio in SSF algorithm.

When the length of the square field is low, the nodes are close together and the detection probability of PUs are high. Hence, it is no difference between PSR algorithm and distance based selection of PUs (SSF or TSF algorithm) as seen in Fig. 7.

## 6. Conclusion

This paper addressed the problem of node selection for sensing and transmission processes and also determining the optimal power of transmission nodes for maximizing the system throughput under some constraints. This optimization problem is solved through convex optimization methods and for implementing it, three algorithms are proposed.

Simulation results show that distance based selection of PUs in SSF and TSF algorithms outperforms the random based selection of PUs in terms of throughput and sensing performance of system. Also, when transmission nodes are selected at first in TSF, we have more choices for transmission node selection, on the other hand there will be more limitation for selecting the sensing nodes. So TSF provides higher throughput than SSF algorithm. But SSF provides higher sensing efficiency and needs lower number of sensing nodes for satisfying the constraint of detection probability. Our solution is independent from the distribution of $P_d$ and can be applied to another fusion rules such as AND rule or "*k*-out-of-*N*" rule.

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