The Study of GENCOs' Bidding Strategies in a Pool-Based Electricity Market Using Cooperative and Non-Cooperative Game Theory

I. Taheri, M. Rashidinejad, A. Badri, and A. Abdollahi

Abstract: Because of the remarkable market share of Generation Companies (GENCOs) in the restructured electricity market, GENCOs competition for supplying electric of power may occur under oligopolistic environment. In such condition, for the sake of maximum profit each GENCO should provide optimal bids. This paper focuses on the short-run bidding behavior of GENCOs under an oligopolistic power market, while the interaction among GENCOs is studied by Game Theory (GT). In case of non-cooperative GENCOs competition, GT proposed Nash equilibrium (NE) as an optimal bidding strategy for each GENCO. On the other hand, GENCOs can make alliances with each other in order to propose their coordinated bids, the so called coalition condition. It can be argued that, the coalition's optimal bidding strategy will be calculated via cooperative GT. Then the obtained profit from such coalition will be allocated among its members based upon Shapley value. In this paper it is assumed that GENCOs submit their bidding blocks in an economic model of supply function equilibrium (SFE). In order to modeling optimal bidding strategy problem of each GENCO, the bi-level programming method is employed in this research. In the upper level, the profit of GENCOs were maximized and in the lower level, the Independent System Operator (ISO) with the aim of minimizing consumers' payment subjected to secured operation of power system, will clear the market. The proposed methodology is implemented on the 30-bus IEEE test system; considering both non-cooperative and cooperative competitions, while GENCOs optimal bidding strategies is calculated. Numerical results show that the efficient alliance has impressive impact on GENCOs profits.

Keywords: Wholesale electricity market, market power, self-interested GENCO, optimal bidding strategy, non-cooperative and cooperative game.

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1. Introduction

In recent years, in order to increase social welfare and to improve market efficiency the supply side of electricity industry became the target of market discipline. In restructured electricity market electrical energy would be traded as a commodity and GENCOs are self-interested agents. Due to limited number of producers, transmission congestion and transmission losses the electricity market is not perfectly competitive than it is more similar to oligopoly. In such environment GENCOs are able to influence the market price through their bids. Thus each GENCO adopts the bidding strategy so that to maximize its profit that derives from wholesale electricity markets [1]. Hence in the restructured power market a profit based bidding decision is a crucial issue for GENCOs. The prior researches on bidding strategies are methodologically classified into three groups. The first group analyzes bidding strategy problem from a perspective of an arbitrary GENCO, based on a pure optimization model by simplifying the rival GENCOs behavior as a set of exogenous variables (stochastic or deterministic). The group of study has developed various mathematical programming models to find an optimal bidding strategy [2-3]. The second group of earlier investigation discusses the bidding strategies based on heuristic models. An evolutionary programming bidding strategy is discussed in [4], while reinforcement learning algorithms are implemented to handle an agent to learn the optimal bidding strategy [5-6]. Finally, the last group of studies discusses the bidding strategies from a viewpoint of GENCOs' behaviors. In wholesale electricity market GENCOs compete with each other to gain higher profit. In [7] the mutual strategic interaction of GENCOs in an imperfect competition is represented by GT. There are two types of games concerning the GENCO's bidding strategy problem, non-cooperative as well as cooperative games. The non-cooperative game is concerned with how GENCOs make decisions when they bid independently. Reference [8] models the bidding strategy problem as a non-cooperative game with complete information. A non-cooperative game with incomplete information is employed in [9] to choose a GENCO's optimal bidding strategy in a restructured power market. In a game of complete information the GENCOs' payoff functions are common knowledge. In a game of incomplete information, in contrast, at least one player is uncertain about another rival GENCO's payoff function.NE and Bayesian NE are the most widely used solution concept respectively for games with complete information and games with incomplete information. However, in cooperative game GENCOs may cooperate with each other to achieve the higher profit than non-cooperative game. Analysis in cooperative GT is centered on two major issues: 1) The selection problem: which coalitions are going to form? 2) The sharing problem: how to allocate profit gained through cooperation such that reflects market power of coalition members [10]. In the first topic the main challenge for each GENCO is to form optimal coalition such that profit of GENCO is maximized. Because the size of different possible coalition formations is exponential in the number of coalition members, the problem of searching for optimal coalition formation is computationally complex. There exist no algorithms in the literature to solve the optimal coalition problem from viewpoint of an arbitrary GENCO in context of wholesale electricity market. In e-market places, existing solutions to this problem is to enumerate some candidate coalitions and select the most profitable alliance. For example [11] provide optimal coalition using an optimal integer partition method. Reference [12] proposed computational study of coalitional games with externalities in the multi-agent system context such that the performance of the entire system is optimized. However, the works on the second topic of coalition is centered on the fairness profit allocation between the members of coalition. Equity-based profit allocation on electricity markets has been scarce. Despite the fact that the GENCOs have different contributions in the profit of the coalition, most of the existing works assumed that the gain of profit in a coalition is spread equally among the coalition members, such as in [13]. In this situation more effective GENCOs have an incentive to deviate from the coalition and eventually intended coalition is failed [14].

In this paper, for modeling security constrained bidding strategy of GENCOs, bi-level programming method is handled. In the upper level of bi-level programming, the profit of GENCOs is maximized and in the lower level, the ISO by considering secure power system operation, clears the market and determines the price that must be paid to GENCOs. Also, an optimal solution for the bi-level programming problem achieves incorporating bid sensitivity functions. Each GENCO profit is influenced by the bidding strategies of rival GENCOs; so GT is used to model the interaction of GENCOs. A non-cooperative game is implemented to find optimal bidding strategies of GENCOs when they bid separately. A cooperative game is utilized to investigate the coalition formation and the bidding strategy problem under coalition. Furthermore the main contribution of this paper is to allocate the profit of coalition among its members based on the Shapley value.

The organization of the rest of the paper is as follows: Section 2 presents the setting an hour-ahead power market for a bidding strategy problem. The proposed solution for finding optimal bidding strategy is presented in Sections 3 and 4. Section 5 provides a case study and illustrates the simulation results of the IEEE30-bus test system. Section 6 provides the conclusion.

2. Electricity Market Setting

A. GENCOs' Bids

Assuming generator j has operation cost function as:

$$C_j\left(P_{gj}\right)=a_j P_{gj}^2+b_j P_{gj}+c_j \tag{1}$$

So marginal cost of GENCO j is:

$$MC_j = 2a_j P_{gj} + b_j \tag{2}$$

MC_j is marginal cost of j_{th} generating unit. Since GENCOs Competition in wholesale electricity market occurs under oligopoly, it is assumed that GENCOs submit their bids in an economic model of SFE. In SFE model, the bid of a GENCO is on the basis of marginal cost, where generally takes one of these two forms: bidding block and continues bid curve**.** In such a case, suppose that generator *j* will submit its bid to ISO as:

$$\rho_j = k_j MC_j = k_j(2a_j P_{gj} + b_j) \tag{3}$$

k_j represents the bidding strategy adopted by the GENCO j. The parameter (k_j) will vary the bid around the true marginal cost curve of the GENCO j where $k_j \geq 1$.

B. Modeling Market Clearing Mechanism

In the pool-based electricity market, ISO may use a load forecasting method to estimate the energy demand for the hour-ahead market. ISO will activate a single-sided uniform price auction for the supply of an inelastic demand, while GENCOs by submit their offers to the ISO, will participate in the auction. Then ISO clears the market and assigns the amount of power each GENCO wins. The process of market clearing can be modeled as a nonlinear optimization problem which the ISO tries to minimize total payment cost based on GENCO's bids. For this purpose, offers are ranked in an increasing order beginning with the least expensive and continuing until the demand is satisfied. On the other hand, since suppliers and consumers are connected through the transmission network, congestion should be considered in the market clearing process. Consequently the process of market clearing can be shown by Eq. (4) [7]:

$$Min \sum_{i=1}^{N_g} \rho_i P_{gi};$$
$$\text{s. t.} \begin{cases} AC\ PowerFlow\ \text{Equations} \\ \theta_i^{ref} \le \theta_i \le \theta_i^{ref};i=n_{ref} \\ v_{\min i} \le v_i \le v_{\max i};i=1,...,N_b \\ P_{gi\min} \le P_{gi} \le P_{gi\max};i=1,...,N_g \\ Q_{gi\min} \le Q_i \le Q_{gi\max};i=1,...,N_g \end{cases} \quad (4)$$

where N_b is the number of buses, N_g is the number of generators, N_l is the number of transmission lines and ρ_j and P_{gj} are bidding price and quantity of generator j, respectively. The variable limits including equality constraints on reference bus angle, limits on the generator real and reactive power, and limits on the voltage magnitude as well transmission line constraint. Since market clearing drives from a non-linear programming problem with nonlinear constraints, in this paper Lagrange method is used to solve the problem. Eq. (4) can be rewritten as follows [7]:

$$\underset{t}{Min}\ f(t);\ where\ \ t=\left[\theta\ v\ \ P_{gi}\ \ Q_{gi}\right]^T$$
$$\text{s. t.}: \begin{cases} p(t)=0 \\ q(t)\le 0 \end{cases} \quad (5)$$

The optimization vector t for the AC OPF problem consists of the N_g vectors of generator real and reactive power injections P_g and Q_g, and the N_b vectors of voltage angles Θ and magnitudes v. In (5), the equality constraints and inequality constraints have been embedded into $p(t)$ and $q(t)$. Vector of positive slack variables s is used to transform the N_{ineq} inequality constraints into equality constraints by incorporating them to logarithmic barrier function. The Lagrangian can be written by Eq. (6) [7]:

$$L^{\gamma}(t,s,\lambda,\mu)=f(t)+\lambda^T p(t)+$$
$$\mu^T(q(t)+s)-\gamma\sum_{m=1}^{N_{ineq}}\ln(s_m) \quad (6)$$

As the parameter of perturbation γ approaches zero, the solution to this problem approaches to the original problem. The necessary conditions for an extreme value of the objective function result when one takes the first derivative of the Lagrange function with respect to each independent variable and set the derivatives equal to zero (Karush-Kuhn-Tucker optimality conditions). Applying first order optimality conditions, K.K.T. equations will be as follows [7]:

$$\frac{\partial L^{\gamma}}{\partial t}=f_t+\lambda^T p_t+\mu^T q_t=0;$$
$$\frac{\partial L^{\gamma}}{\partial s}=\mu^T-\gamma e^T[s]^{-1}=0;$$
$$\frac{\partial L^{\gamma}}{\partial \lambda}=p_{(t)}^T=0;\ \frac{\partial L^{\gamma}}{\partial \mu}=q_{(t)}^T+s^T=0 \quad (7)$$

In this paper, K.K.T. conditions are solved simultaneously by using Newton's method. The Newton updating step can be written as follows:

$$\begin{bmatrix} L_{tt}^{\gamma} & 0 & p_t^T & q_t^T \\ 0 & [\mu] & 0 & [s] \\ p_t & 0 & 0 & 0 \\ q_t & I & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta t \\ \Delta s \\ \Delta\lambda \\ \Delta\mu \end{bmatrix} = -\begin{bmatrix} L_t^{\gamma T} \\ [\mu]s-\gamma e \\ p(t) \\ q(t)+s \end{bmatrix} \quad (8)$$

This set of equations can be simplified by solving explicitly for Δμ in terms of Δs and for Δs in terms of Δt.

$$\Delta\mu=-\mu+[s]^{-1}\left(\gamma e-[\mu]\Delta s\right);$$
$$\Delta s=-q(t)-s-q_t\Delta t; \quad (9)$$

$$\begin{bmatrix} G & p_t^T \\ p_t & 0 \end{bmatrix} \begin{bmatrix} \Delta t \\ \Delta\lambda \end{bmatrix} = -\begin{bmatrix} H \\ p(t) \end{bmatrix};$$
$$\begin{cases} G\equiv L_{tt}^{\gamma}+q_t^T[s]^{-1}[\mu]q_t \\ H\equiv L_t^{\gamma T}+q_t^T[s]^{-1}\left(\gamma e+[\mu]q(t)\right) \end{cases} \quad (10)$$

In order to satisfy the first order optimality conditions of the original problem during the Newton iterations, the perturbation parameter γ must converge to zero. In this paper the γ is updated by:

$$\gamma^{new}=\sigma\frac{s^T\mu}{N_{ineq}} \quad (11)$$

In (11), σ is a number between 0 and 1. When ISO clears the market, each GENCO will be paid based upon Locational Marginal Price (LMP). The LMP at each bus is the Lagrangian multiplier of the corresponding AC power flow constraint. So, the profit of GENCO j is:

$$R_j(k_j,k_{-j})=P_{gj}*LMP_j-C_j\left(P_{gj}\right) \quad (12)$$

where P_{gj} is the power produced by GENCO j during one hour, and $C_j(P_{gj})$ is the cost of producing P_{gj} (MWh) [7].

In (12), k_j represents the strategies of GENCO j and k_j those of its competitors.

3. Bidding Strategies under Non-Cooperative Competition

A. GENCO Profit Maximization

Due to the incomplete nature of competition in the wholesale electricity market, generation scheduling and market clearing price will be affected by the bids of GENCOs that have privilege of market power. In oligopolistic market GENCOs are price makers and they are called strategic players, thereby strategic players drive up the price enough to increase their profit. Prices can be influenced by raising the asking price. In this case, each GENCO will have to select the optimal bidding strategy to achieve maximum profits. In order to determine optimal bidding strategy, GENCO j faced with the following problem [7]:

$$\begin{aligned} &\underset{k_j}{Max} \quad R_j(P_{gj}, LMP_j) = P_{gj} * LMP_j - C_j\left(P_{gj}\right) \\ &\text{s. t.} \quad \begin{cases} k_{j\,\min} < k_j < k_{j\,\max} \\ \text{s. t.: } \ equation(4) \end{cases} \end{aligned} \tag{13}$$

In (13), to modeling optimal bidding strategy problem of an arbitrary GENCO j, in the upper level of bi-level programming the profit of GENCO j are maximized and in the lower level the ISO with the aim of minimizing consumers' payment subjected to secured operation, will clear the market. In this paper, the optimal bidding strategy of GENCO j is calculated based on an iterative method, where it derives the optimal bidding strategy developed in (14). It starts with and initial strategy for the GENCO j and using the sensitivity function, after few iterations the optimal bidding strategy of GENCO j will be determined. This paper uses the following rule Eq. (14) to update the bidding strategy of GENCO j:

$$k_j^{new} = k_j^{old} + \alpha \frac{\partial R_j}{\partial k_j^{old}} \tag{14}$$

where α is a constant for controlling the iteration step of k_j, and $(\partial R_j / \partial k_j^{old})$ representing the sensitivity of profit to the bidding strategy. At optimal bidding strategy k_j^*, the sensitivity of profit to trivial perturb in strategy is equal to zero [9].

B. Finding Nash Equilibrium Point

With regard to the upper level Eq. (13), j_{th} GENCO profit is a function of its quantity and price of power sold. On the other hand, at lower level ISO will determine the generation scheduling and price that must be paid to each GENCO. Therefore, the profit of each GENCO is implicitly a function of the strategies adopted by all GENCOs. This means that GENCO j cannot optimize its profits by its own, where it must consider what the other GENCOs will do. However, it is reasonable to make the assumption that all GENCOs are behaving in a rational manner, that is, all GENCOs trying to maximize their profits. Thus, determining optimal bidding strategy of GENCO j can be considered an interactive optimization problem. In this section determining optimal bidding strategy discusses as a non-cooperative game. The solution of a non-cooperative game is called NE and represents market equilibrium under oligopolistic environment [15]. In a non-cooperative game the strategies $(k_1^*, \ldots, k_j^*, \ldots, k_{N_g}^*)$ are a NE, for each GENCO j, k_j^* is j_{th} GENCO best response to the strategies specified for the N_g-i other GENCOs, $(k_1^*, \ldots, k_{j-1}^*, k_{j+1}^* \ldots, k_{N_g}^*)$. So, k_j^* solves:

$$\begin{aligned} &\underset{k_j}{\text{Max}} \quad R_j(k_j; k_{-j}^*) = \\ &R_j(k_1^*; \ldots; k_{j-1}^*; k_j; k_{j+1}^*; \ldots; k_{N_g}^*) \end{aligned} \tag{15}$$

According to Eq. (15), the simultaneous solving Eq. (13) for all GENCOs have participated in the market, the NE is obtained. In this paper it is assumed that each GENCO's profit is commonly known to all GENCOs. Thus for the sake of simplicity a game with complete information is used to determine optimal bidding strategy of GENCOs. The methodology flow diagram that is employed for solving a game with complete information is shown in Fig. 1.

4. Bidding Strategies under Coalition

Coalition may happen if a potential for higher profit is promising, where it is discussed in the following.

A. Coalition for Profit Maximization

In a wholesale electricity market NE is not necessarily Pareto optimal, denoting that there are market equilibriums, which are more profitable for all the players, than NE ones [16]. A Pareto optimal outcome cannot be improved upon without losing at least one player. In an n-person game the set of feasible strategies $(k_1^p, \ldots, k_{Ng}^p)$ is Pareto optimal if there does not exist another feasible strategies $(\tilde{k}_1, \ldots, \tilde{k}_{Ng})$ such that:

$$\begin{aligned} &R_1(k_1^p, \ldots, k_{Ng}^p) \le R_1(\tilde{k}_1, \ldots, \tilde{k}_{Ng}) \\ &\qquad \vdots \\ &R_{Ng}(k_1^p, \ldots, k_{Ng}^p) \le R_{Ng}(\tilde{k}_1, \ldots, \tilde{k}_{Ng}) \end{aligned} \tag{16}$$

with at least one of the above inequalities is strict. Whoever

GENCOs such adopts their bidding strategies that the market equilibrium be Pareto optimal, thus in an oligopolistic electricity market GENCOs may have an incentive to cooperate. The term cooperation refers to GENCOs interacting with a common purpose. GENCOs strategies in a cooperative game strongly Pareto dominates NE if the profit of each player in case of cooperation is higher than the profits when bidding separately belong non-cooperative game. That is, in this section determining optimal bidding strategy discussed as a cooperative game. Let $\Gamma = (N, v)$ be an n-person game with transferable utility (a TU game) in coalition form where $N = \{1, \ldots, N_g\}$ is set of players and v is the characteristic function (coalition function).A group of players who cooperate with each other are said to form a coalition (A coalition is every non-empty subset $S \subseteq N$ of cooperating players). The characteristic function assigns the utility of a coalition. Since the number of all subsets of N is 2^{N_g}, consequently $v : 2^{N_g} \to \mathbb{R}$. In this paper the characteristic function is common knowledge and assigns a profit value to any coalition of a few GENCOs. In an oligopoly market with N_g participants, various combinations of coalitions possibly may occur. Here, it is assumed the number of combinations of all potential coalitions is N_c. An arbitrary combination of potential coalition Ψ_{nc} is represented by a coalition structure. The collection of all coalition structures in N is denoted by $\Psi = \{\Psi_1, \ldots, \Psi_{Nc}\}$ .A coalition structure (CS) $\Psi_{nc} = \{\mathfrak{P}_{nc}^1, \ldots, \mathfrak{P}_{nc}^{Np}\}$ of combination Ψ_{nc}, is a partition of N where $\mathfrak{P}_{nc}^{np} = \{\mathfrak{g}_{nc}^{np}(1), \ldots, \mathfrak{g}_{nc}^{np}(\Omega)\}$ is a coalition with Ω GENCOs, $1 \le \Omega < Ng$. The number of GENCOs in $\mathfrak{P}_{nc}^{np}$ is also known as $\mathfrak{P}_{nc}^{np}$'s cardinality, $\|\mathfrak{P}_{nc}^{np}\| = \Omega$. $\|\mathfrak{P}_{nc}^{np}\| = 1$'s means that in partition $\mathfrak{P}_{nc}^{np}$ one GENCO bids single-handedly in contradiction of rival coalitions (partitions) $\{\mathfrak{P}_{nc}^{-np}\} = \Psi_{nc} - \{\mathfrak{P}_{nc}^{np}\} = \{\mathfrak{P}_{nc}^1, \ldots, \mathfrak{P}_{nc}^{np-1}, \mathfrak{P}_{nc}^{np+1}, \ldots, \mathfrak{P}_{nc}^{Np}\}$. In an arbitrary combination Ψ_{nc}, $\bigcup_{np=1}^{Np} \mathfrak{P}_{nc}^{np} = N$ and $\mathfrak{P}_{nc}^{np} \cap \mathfrak{P}_{nc}^{nq} = \phi; \mathfrak{P}_{nc}^{np} \subseteq \Psi_{nc}, \mathfrak{P}_{nc}^{nq} \subseteq \Psi_{nc}$. For a potential combination Ψ_{nc}, cooperative game is applied in each partition $\mathfrak{P}_{nc}^{np}$. In a cooperative game, players adopt strategies that result in the best consequence for a coalition. $R_{\mathfrak{g}_{nc}^{np}(\omega)}$ represents the profit of a GENCO ω belonging to the coalition $\mathfrak{P}_{nc}^{np}$ in the coalition structure Ψ_{nc}. To find an optimal bidding strategy of GENCO $\mathfrak{g}_{nc}^{np}(\omega)$ [13]:

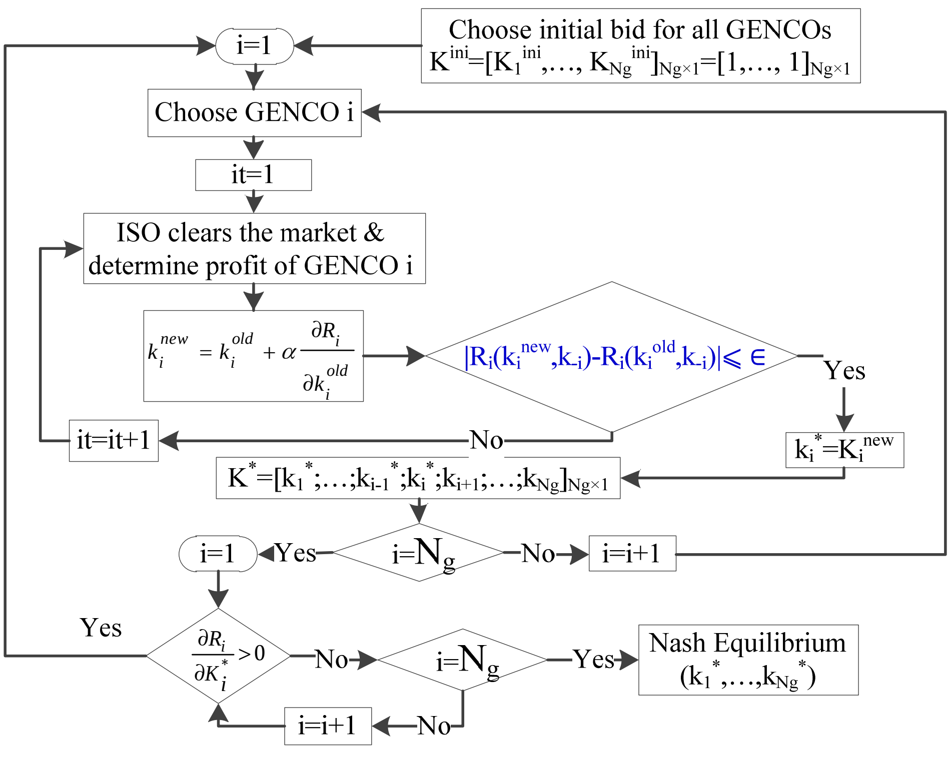

Fig. 1. Solution of non-cooperative game.

$$\underset{k_{\mathfrak{g}_{nc}^{np}(\omega)}}{Max}\ \mathrm{R}_{\mathfrak{P}_{nc}^{np}} = \sum_{\omega=1}^{\left\|\mathfrak{P}_{nc}^{np}\right\|}\left\{P_{\mathfrak{g}_{nc}^{np}}(\omega) * LMP_{\mathfrak{g}_{nc}^{np}}(\omega)\right\} - C_{\mathfrak{g}_{nc}^{np}}(\omega)\left(P_{\mathfrak{g}_{nc}^{np}}(\omega)\right)$$

$$s.t.: \begin{cases} (k_{\mathfrak{g}_{nc}^{np}(\omega)})\min < k_{\mathfrak{g}_{nc}^{np}}(\omega) < (k_{\mathfrak{g}_{nc}^{np}(\omega)})\max \\ Min \displaystyle\sum_{np=1}^{Np}\sum_{\omega=1}^{\left\|\mathfrak{P}_{nc}^{np}\right\|}\left(\rho_{\mathfrak{g}_{nc}^{np}(\omega)} * P_{\mathfrak{g}_{nc}^{np}(\omega)}\right); \end{cases}$$

$$s.t.: \begin{cases} AC\ PowerFlow\ \text{Equations} \\ \theta_i^{ref} \le \theta_i \le \theta_i^{ref}\ ; i = n_{ref} \\ v_{\min i} \le v_i \le v_{\max i}\ ; i = 1,\dots,N_b \\ P_{gi\ \min} \le P_{gi} \le P_{gi\ \max}\ ; i = 1,\dots,N_g \\ Q_{gi\ \min} \le Q_i \le Q_{gi\ \max}\ ; i = 1,\dots,N_g \\ F_{l\min} \le F_l \le F_{l\ \max}\ ; l = 1,\dots,N_l \end{cases} \tag{17}$$

Equation (18) describes an iterative technique for finding optimal bidding strategy $k^*_{np_{\mathfrak{g}_{nc}(\omega)}}$ of an arbitrary GENCO $\mathfrak{g}_{nc}^{np}(\omega)$ in (17).

$$k^{new}_{\mathfrak{g}_{nc}^{np}(\omega)} = k^{old}_{\mathfrak{g}_{nc}^{np}(\omega)} + \alpha\frac{\partial \mathrm{R}_{\mathfrak{P}_{nc}^{np}}}{\partial k^{old}_{\mathfrak{g}_{nc}^{np}(\omega)}} \tag{18}$$

The profit of a coalition depends on strategies taken by rival coalition. In this case, non-cooperative GT is used for modeling coalitions in an oligopolistic competition. The algorithm illustrated in Fig. 1is employed for solving coalition competition.

B. Profit Allocation

The value of two disjoint coalitions is at least as great when they work together as when they work apart. This property is captured in the following definition of super-additivity.

$$\begin{aligned} &If\ \ S \cap T = \phi, S \subseteq N, \quad T \subseteq N; \\ &then\ \ v(S) + v(T) \leqslant v(S \cup T) \end{aligned} \tag{19}$$

Although super-additivity seems to be a natural assumption, but in a wholesale electricity market the assumption of super-additivity is not necessarily established. Thus it is possible in a particular combination Ψ_{nc} some GENCOs harm due to join in a coalition $\mathfrak{P}_{nc}^{np}$. In this case coalition $\mathfrak{P}_{nc}^{np}$ is not stable. In this study generation cost differences will affect the stability of coalition. If GENCOs have different marginal costs, they will have different ideas about the strategy that makes maximum profit. Therefore a coalition is more likely to be stable if its members have similar cost functions. Indeed in this paper, to assess the stability of a coalition based on the principle of super-additivity a technique is developed that is applied ex-ante (looking for the potential of stability of a coalition). According to the aforementioned description, full load average cost (FLAC) of each GENCO has been introduced as a marker for assess the ex-antestability of a coalition [17]. The FLAC of GENCO j is calculated via Eq. (20):

$$FLAC_j = \frac{C_j\left(P_{gj}\max\right)}{P_{gj}\max} \tag{20}$$

It is assumed that, if the difference between GENCOs FLAC is high, their coalition may fail, otherwise there is a potential for coalition. Once GENCOs decide to cooperate together, then the problem is how to allocate or distribute the total profit among the dissimilar members of the coalition (the term 'dissimilar member' refers to the different contributions of GENCOs to the coalition's value). Furthermore, every participant desires to attain its maximum profit in the coalition. The reasonable schemes of profits allocation in the coalition are known as solution concepts and they are based on disparate interpretations of fairness. In favor of a stable allocating that satisfies fairness criterion, several solution concepts are provided, such as the core, Shapley value, bargaining set, stable set, nucleolus, and kernel [14].The idea of stability in cooperative GT corresponds to the idea of NE in non-cooperative GT. In non-cooperative GT, NE is a situation such that no individual can deviate and make higher profit. In cooperative GT, a stable allocation is a situation such that no coalition can deviate and make its members better off. In this paper for the profit allocation among GENCOs $\mathfrak{g}_{nc}^{np}(1),\dots,\mathfrak{g}_{nc}^{np}(\Omega)$ in the coalition $\mathfrak{P}_{nc}^{np}$, the Shapley value is used. Intuitively, the Shapley value captures the expected marginal contribution of GENCO $\mathfrak{g}_{nc}^{np}(\omega)$ over all the possible orders. Number of all orders (permutations) of GENCOs on coalition $\mathfrak{P}_{nc}^{np}$ is $\Omega!$. The set of all permutations on $\mathfrak{P}_{nc}^{np}$ is represented by $SoP(\mathfrak{P}_{nc}^{np})$. The function $RO_{\mathfrak{P}_{nc}^{np}} : \mathfrak{P}_{nc}^{np} \to \mathfrak{P}_{nc}^{np}$ is called rank orders on $\mathfrak{P}_{nc}^{np}$. Also $\zeta^{\mathfrak{P}_{nc}^{np}}_{\mathfrak{g}_{nc}^{np}(\omega)}(RO_{\mathfrak{P}_{nc}^{np}})$ is indicated the set of all GENCOs up to and including GENCO $\mathfrak{g}_{nc}^{np}(\omega)$ under rank order $RO_{\mathfrak{P}_{nc}^{np}}$ [18].

$$\begin{aligned} &\zeta_{\mathfrak{g}_{nc}^{np}(\omega)}(RO_{\mathfrak{P}_{nc}^{np}}) = \{RO_{\mathfrak{P}_{nc}^{np}}(1),\dots,RO_{\mathfrak{P}_{nc}^{np}}(n_{ro})\}; \\ &RO_{\mathfrak{P}_{nc}^{np}}(n_{ro}) = \mathfrak{g}_{nc}^{np}(\omega) \end{aligned} \tag{21}$$

Thus, $\zeta_{\mathfrak{g}_{nc}^{np}(\omega)}(RO_{\mathfrak{P}_{nc}^{np}})$ is the set of GENCOs who enters in the rank order $RO_{\mathfrak{P}_{nc}^{np}}$ after GENCO $\mathfrak{g}_{nc}^{np}(\omega)$ has entered [18]. GENCO $\mathfrak{g}_{nc}^{np}(\omega)$'s marginal contribution with regard to rank order $RO_{\mathfrak{P}_{nc}^{np}}$ is signified by $\mathfrak{MC}^{RO_{\mathfrak{P}_{nc}^{np}}}_{\mathfrak{g}_{nc}^{np}(\omega)}$ and is specified by:

$$\begin{aligned} \mathfrak{MC}^{RO_{\mathfrak{P}_{nc}^{np}}}_{\mathfrak{g}_{nc}^{np}(\omega)} &= \boldsymbol{v}(\zeta_{\mathfrak{g}_{nc}^{np}(\omega)}(RO_{\mathfrak{P}_{nc}^{np}})) - \\ &\boldsymbol{v}\left(\zeta_{\mathfrak{g}_{nc}^{np}(\omega)}(RO_{\mathfrak{P}_{nc}^{np}}) \setminus \mathfrak{g}_{nc}^{np}(\omega)\right) \end{aligned} \tag{22}$$

The Shapley value (Sh) is the solution function for fairness allocation of profit, and known by:

$$Sh_{\mathfrak{g}_{nc}^{np}(\omega)} = \frac{1}{\Omega!} \sum_{{}^{RO}\mathfrak{P}_{nc}^{np}=1}^{n_{ro}} \mathfrak{MC}_{\mathfrak{g}_{nc}^{np}(\omega)}^{{}^{RO}\mathfrak{P}_{nc}^{np}} \tag{23}$$

The Shapley value is the average of all marginal profits which GENCO $\mathfrak{g}_{nc}^{np}(\omega)$ contributes to coalition.

5. Case Study and Simulation Results

The IEEE-30 bus test system is employed to implement the proposed methodology by illustrating and analyzing simulation results. It is assumed that there are six GENCOs (each containing one unit), competing with each other. Here it is assumed that demand is fixed and inelastic. The cost coefficients and FLAC of generators are given in Tables 1. Since the fixed costs (c) are constant, they do not contribute to the marginal cost. Table 2 illustrates GENCOs' outputs, corresponding payoffs and MCP in fully competitive market.

Table 1. Generators' data.

Gen. Buses	A	b	P_{gmax} (MW)	FLAC ($/h)
1	0.02	2	120	4.4
2	0.175	1.75	60	12.25
3	0.625	1	90	57.25
4	0.0834	3.25	70	9.088
5	0.25	3	80	23
6	0.25	3	80	23

Table 2. Generation outputs, MCP and profit in competitive electricity market.

Gen. Buses	k_i	Output (MWh)	MCP ($/MWh)	Profit($/h)
1	1	119.9997	7.814565	409.7475
2	1	17.32494	7.814565	52.54135
3	1	5.424906	7.814565	18.57487
4	1	27.46125	7.82573	62.76163
5	1	9.454376	7.814565	23.1724
6	1	9.534788	7.814565	23.17781

For detection the potential of market power, two structural measures as the market share concentration ratio and Herfindahl-Hirschman Index (H.H.I.) are applied to competitive an hour-ahead power market [19]. In the power market, the percentage of market share of the largest GENCO is the market share concentration ratio. If market share concentration ratio exceeds 20%, it will denote the potential of exercising market power. The H.H.I. index is a well-known concentration measure that is calculated by summing the squares of the market shares of all individual market participants. The US Department of Justice/Federal Trade Commission standards divides the spectrum of market concentration as measured by the H.H.I. into three regions that can be broadly characterized as un-concentrated (H.H.I. below 1000), moderately concentrated (H.H.I. between 1000 and 1800), and highly concentrated (H.H.I. above 1800) [20]. Market share and H.H.I of competitive electricity market are presented in Table 3, respectively. According to Tables 3, the competitive electricity generation market is highly concentrated, where under such conditions; GENCOs are able to exercise market power.

Table 3. Market share and H.H.I. in competitive electricity market.

Gen. Buses	Market Share	H.H.I.
1	0.63424802	4375.8
2	0.09156947	
3	0.028672871	
4	0.145144058	
5	0.049970285	
6	0.050395296	

In a non-cooperative competition, for applying optimal market power, bidding strategies of GENCOs are chosen based on the NE. The results of GENCOs competition under oligopoly market are presented in Table 4.The GENCOs' profit in the oligopoly market and competitive market are compared in Fig.2.According to Fig. 2, in oligopoly power market the profits of all GENCOs have been increased. Comparing Tables 2 and 4 indicate that dominant generator (generator 1 with 63% market share) increased MCP through economic withholding. So rests GENCOs (GENCOs 2 to 6) have opportunity to sell more power in imperfect competitive market. Therefore, as the result of GENCOs interactions in imperfect competition, GENCOs profits have increased in comparison with the fully competitive market. Also GENCOs for optimal exercise of market power can cooperate to form some coalitions. When coalition is taken into account, the number of combinations of all potential coalitions is 720=6!.

Table 4. Generation outputs, MCP and profit in oligopolistic electricity market.

Gen. Buses	Optimum k_i	Output (MWh)	MCP ($/MWh)	Profit ($/h)
1	2.37	80.53	12.383	706.47
2	1.136	26.127	12.38	158.601
3	1.129	7.93	12.383	50.97
4	1.225	41.13	12.383	234.81
5	1.122	16.096	12.383	86.26
6	1.058	17.39	12.383	87.58

For the sake of simplicity, in this case study optimal bidding strategy of GENCOs calculated for three combination of potential coalition: scenario1: {(1), (2-3), (4), (5), (6)}, scenario2: {(1), (3), (2-4), (5), (6)}, scenario3:{(1), (2-3-4), (5), (6)}. Table 5 shows GENCOs' outputs, related payoffs and MCP for scenario1. Fig. 3compares Tables 4 and 5, it can be seen that the profit of GENCOs 2 and 3 is decreased when these two GENCOs make a coalition. Therefore coalition of GENCOs 2 and 3 is unstable. This can be described due to large difference between FLAC of GENCOs 2 and 3 (see the Table 1). Also the profit of GENCOs 2 and 3 based on Shapley allocation are presented in Table 6.

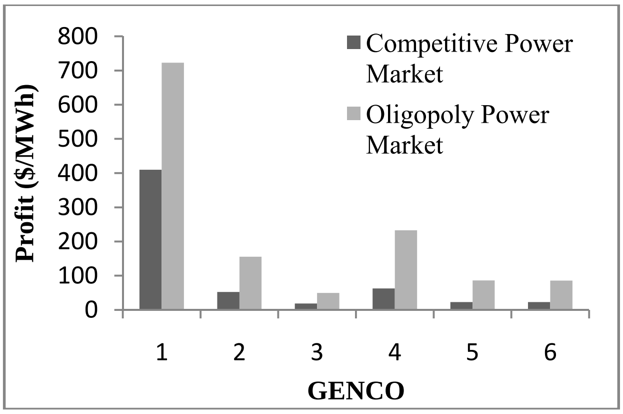

Fig. 2. GENCOs profit of competitive and non-cooperative oligopoly power market.

Table 5. Generation outputs, MCP and profit in scenario 1.

Gen. Buses	k_i	Output (MWh)	MCP ($/MWh)	Profit ($/h)
1	2.32	83.12662	12.3594	722.9412
2	1.18	24.88476	12.34659	155.3247
3	1.23	7.210487	12.38034	49.56333
4	1.23	40.72105	12.35706	232.5548
5	1.08	16.81881	12.3427	86.41505
6	1.09	16.43828	12.32831	85.78717

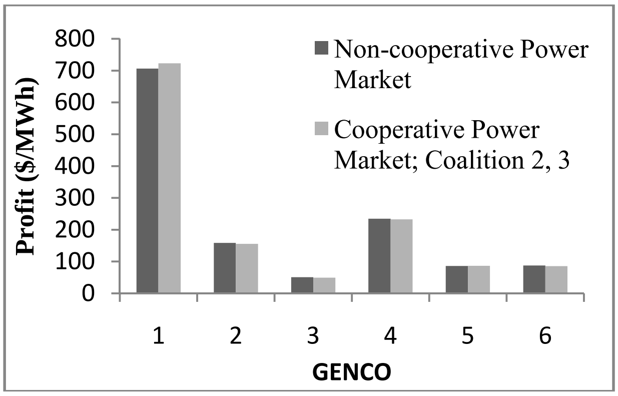

Fig. 3. GENCOs profit of non-cooperative and cooperative in oligopoly power market.

Table 6. Comparison of profits before and after cooperation 2, 3.

Gen. Buses	Non-cooperative	Cooperative(Shapley Value)
2	158.601	156.2595
3	50.97	48.62849

Table 7 shows GENCOs' outputs, corresponding payoffs and MCP for scenario2. Comparing generation scheduling and bidding strategies of GENCOs 2 and 3 in Tables 4 and 7 indicate that coalition of GENCOs 2 and 4 increased MCP through economic withholding or physical withholding. The total profit of the GENCOs 2 and 4 in Fig. 4 show that profit of the GENCOs 2 and 4 are greater than without the coalition. Also the profit of GENCOs 2 and 4 based on Shapley allocation are presented in Table 8.

Table 7. Generation outputs, MCP and profit in scenario 2.

Gen. Buses	k_i	Output (MWh)	MCP ($/MWh)	Profit ($/h)
1	2.44	85.79711	13.2566	818.5609
2	1.44	21.17774	13.22143	164.452
3	1.12	8.587673	13.22143	58.86105
4	1.4	37.1746	13.23045	255.7644
5	1.09	18.12139	13.22143	103.1303
6	1.08	18.34148	13.22143	103.3737

Table 8. Comparison of profits before and after cooperation 2, 4.

Gen. Buses	Non-cooperative	Cooperative (Shapley Value)
2	158.601	172.0037
4	234.81	248.2127

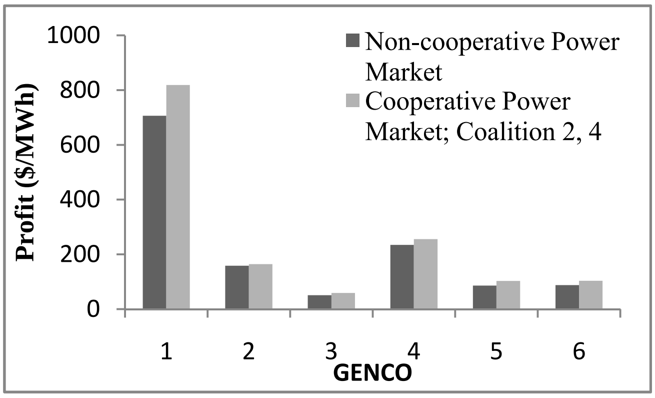

Fig. 4. GENCOs profit of non-cooperative and cooperative in oligopoly power market.

In the scenario 3, GENCOs 2, 3 and 4 have coalition. Table 9 shows GENCOs' outputs, corresponding payoffs and MCP for scenario 3. Comparing bidding strategies of GENCOs 2, 3 and 4 in Tables 4 and 9 show that coalition of GENCOs 2, 3 and 4 have asked higher price for selling power. So GENCO 1 has opportunity to sell power in higher price. GENCOs interaction eventually caused MCP form 12.38 ($/MWh) in non-cooperative competition increase to 14.56 ($/MWh) in cooperative competition. So GENCOs 5 and 6 have opportunity to sell more power. In according to Fig. 5, the profit of all GENCOs under coalition of GENCOs 2, 3 and 4 is greater than the GENCOs profit without the coalition. The profit of GENCOs 2, 3 and 4 based on Shapley allocation are presented in Table 10.

Table 9. Generation outputs, MCP and profit in scenario 3

Gen. Buses	k_i	Output (MWh)	MCP ($/MWh)	Profit ($/h)
1	2.67	86.36256	14.55751	935.3292
2	1.59	21.16188	14.56426	192.8044
3	1.61	6.399572	14.5734	61.26738
4	1.58	35.86933	14.58064	299.1192
5	1.14	19.46936	14.54468	130.0036
6	1.12	19.9373	14.54468	130.7959

Table10. Comparison of profits before and after cooperation 2, 3, 4

Gen. Buses	Non-cooperative	Cooperative (Shapley Value)
2	158.601	178.5078
3	50.97	56.91578
4	234.81	256.5

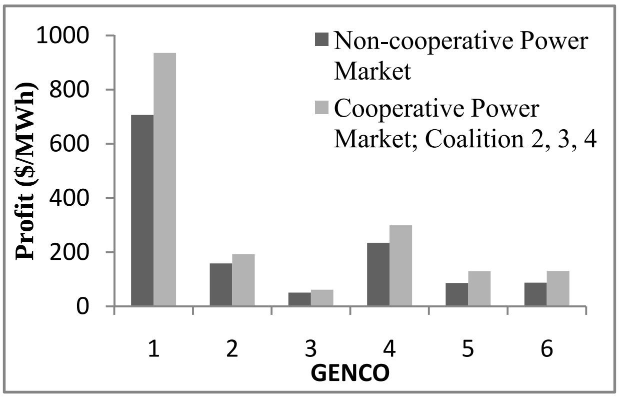

Fig. 5. GENCOs profit of non-cooperative and cooperative in oligopoly power market

6. Conclusion

In this paper GENCOs bidding strategies in the context of an oligopoly market has been studied. GENCOs competition with each other investigated on the basis of the economic equilibrium model of SFE. In the market equilibrium no participant can improve its profit by unilaterally deviating from its offer. The main focuses of this paper is on GENCOs optimal bidding strategies in oligopolistic market, possibility of the cooperation of GENCOs, and fairly allocate the profit among the members of a coalition based on Shapley value. The proposed methodology in this paper is implemented on the 30-bus IEEE test system. To compare GENCOs' equilibrium behavior under different levels of market competitiveness; we compare the results from perfect competition, static Nash oligopoly and under coalition. Case study indicates in oligopoly marketplace GENCOs' profits are increased in comparison with a perfect competition market due to optimal bidding strategies. Also it is shown that in cooperative game GENCOs can achieve higher profits than in NE and by allocating profit based on Shapley value the stability are maintained for two of the three coalitions.

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